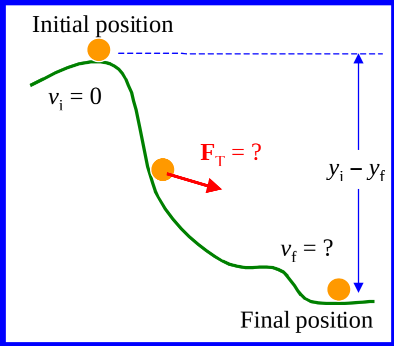
Work and Energy

Newton's Second Law and Conserved Quantities

• Newton's second law sometimes cannot be applied directly since the force is unknown or difficult to calculate, e.g., a ball rolling down an irregular frictionless hill.

• However, from Newton's second law, we know that some quantities describing an isolated physical system do not change their values with time, i. e., conserved quantities. Problems, such as this example, can be solved easily using them.



• Three main conserved quantities in mechanics: energy, momentum and angular momentum.

• If properly defined, these conserved quantities can be extended to other systems: e. g., chemical energy, thermal energy.

• It requires external work (work done) to change the energy of a system.

Work Done (Work)

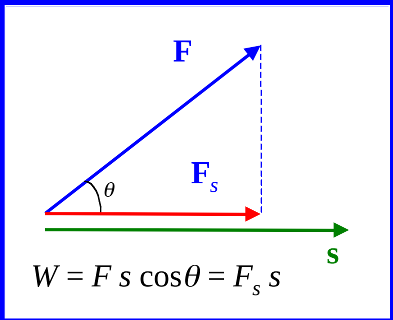
Work done by a constant force F over a displacement s

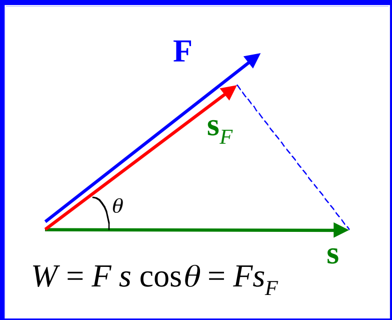
• Work done is a scalar defined by $W = F\, s \cos\theta$

F: magnitude of the constant force

s: magnitude of the displacement that F has acted on the object

θ : angle between the vectors **F** and **s**





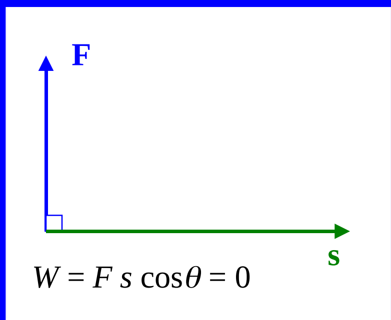
• $W = s$ times component of **F** along the direction of **s**

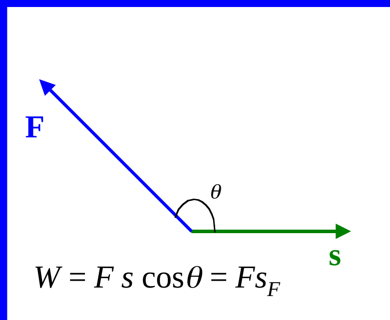
• $W = F$ times component of **s** along the direction of **F**

Work Done (Work)

Work done by a constant force F over a displacement s

- Work done can be zero is θ =90 degrees so that $W = F\, s \cos 90° = 0$
  i. e., the vectors **F** and **s** are perpendicular to each other.
- Work done can be negative, if $90° < \theta < 270°$.
- SI unit of work done: Joule (J = N m).





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Work and Kinetic Energy

Work - Energy Theorem

- Consider the case with a constant force component along a 1D motion.
- Use the equation: $v^2 = v_0^2 + 2ax$
- With $x = s$, $F_s = m\, a$, $W = F_s s = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$
- Define kinetic energy of an object with mass m with speed v:

$$\text{KE} = \frac{1}{2}mv^2 \qquad \text{(SI unit: J)}$$

- Work-Energy theorem: $W = \text{KE}_\text{f} - \text{KE}_0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$
- Also valid for a non-constant force.
- KE is conserved if the work done W by the external force is zero.
- The object does the work (and slow down) if $W < 0$.

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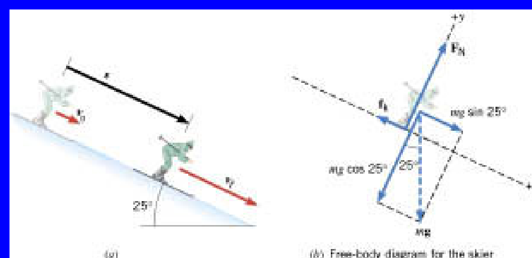
Work and Kinetic Energy

Work - Energy Theorem (Example 5)

- Downhill skiing: m = 58 kg, θ = 25º, f_k = 70 N, v_0 = 3.6 m/s, s = 57 m, v_f = ?
- Method 1: $a = \frac{F}{m} = \frac{mg\sin\theta - f_\mathrm{k}}{m} = 2.935\ \mathrm{m/s^2}, \quad v_f = \sqrt{v_0^2 + 2as} = 18.64\ \mathrm{m/s}$
- Method 2: $W = Fs = (mg\sin\theta - f_\mathrm{k})s = 9702\ \mathrm{m/s^2} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2,$

  $v_f = \sqrt{\frac{2W}{m} + v_0^2} = 18.64\ \mathrm{m/2}$



Work and Gravitational Potential Energy

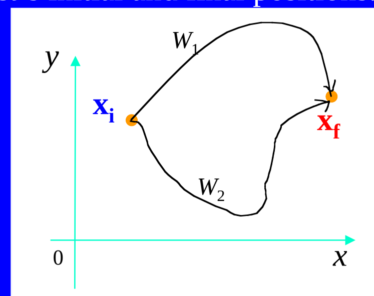
Work Done by the force of gravity

- Gravitational force near the surface of the Earth: $\mathbf{F}_g = -\mathrm{mg}\,\hat{\mathbf{y}}$
- y-component of displacement $\mathbf{s} = s_y = h_f - h_0$
- $W = F_s\, s = -m\, g\, s_y = m\, g\, (h_0 - h_f)$
- Define gravitational potential energy of an object with mass m at hight h relative to an arbitrary zero level: $\boxed{\mathrm{PE} = mgh}$ (SI unit: J)
- Work done by gravity: $\boxed{W = \mathrm{PE}_0 - \mathrm{PE}_\mathrm{f} = mg(h_0 - h_\mathrm{f})}$
- Only difference between PE is important.
- Energy stored in PE (object gets up) if $W < 0$.

Work and Energy

Conservative and Non-Conservative Forces

- "A force is conservative when the work it does on a moving object is independent of the path between the object's initial and final positions.
- No work done around a close path, since $W = W_1 - W_2 = 0$
- Potential energy depending only on position can be defined for conservative force.
- Gravity is a conservative force.
- Friction is a non-conservative force.
- $W_{nc} = (KE_f - KE_0) + (PE_f - PE_0) = (KE_f + PE_f) - (KE_0 + PE_0)$



$$W_{nc} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2 + mg(h_f - h_0)$$

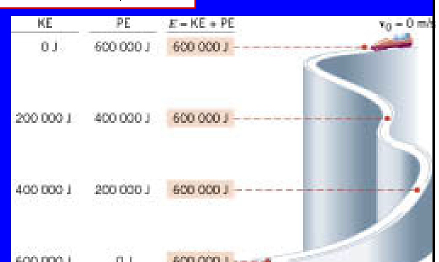
Work and Energy

Conservation of Mechanical Energy

- Total mechanical energy: $E = KE + PE$
- Conservation if $W_{nc} = (KE_f + PE_f) - (KE_0 + PE_0) = E_f - E_0 = 0$, or

$$W_{nc} = \left(\tfrac{1}{2}mv_f^2 + mgh_f\right) - \left(\tfrac{1}{2}mv_0^2 + mgh_0\right) = 0$$

- Total energy remains constant if no work done by non-conservative force.
- Can be extended to include other forms of energy: heat, electrical, chemical, sound, nuclear, etc.



Work and Energy

Conservation of Mechanical Energy (Example 8)

- v_0 = 38 m/s, h_0 = 70 m, h_f = 70 m, v = ?
- Method 1: $v_x = v_{0x} = 38.0$ m/s, $v_y = \sqrt{2g(h_0 - h_f)} = 26.19$ m/s, $v = 46.2$ m/s
- Method 2:

$$E_f = \tfrac{1}{2}mv^2 + mgh_f = E_0 = \tfrac{1}{2}mv_0^2 + mgh_0$$

$$v = \sqrt{v_0^2 + 2g(h_0 - h_f)} = 46.2 \text{ m/s}$$

- Can be applied to more complicated situation, e. g., Example 10 -- The Steel Dragon, which can be solved by Method 2 (conservation of energy) but not by Method 1 (kinematics equations).

Do cq6-7