# Circular Motion

**Uniform Circular Motion**

• An object traveling at a constant speed $v$ (not velocity) on a circular path.

• Period $T$ is the time required to travel once around the circle with a radius $r$ (one revolution): $T = 2\,\pi\, r\,/\,v$ , or $v = 2\,\pi\, r\,/\,T$

• Frequency $f = 1/T = v\,/\,2\,\pi\, r$ , or $v = 2\,\pi\, r\, f$

• Unit of frequency: revolution/s (or 1/s) or revolution/minute (rpm)

• Example: frequency of the rotation of the Earth

$f_E$ = 1 revolution/24 hours = $6.94\times10^{-4}$ rpm or $1.16\times10^{-5}$ s$^{-1}$

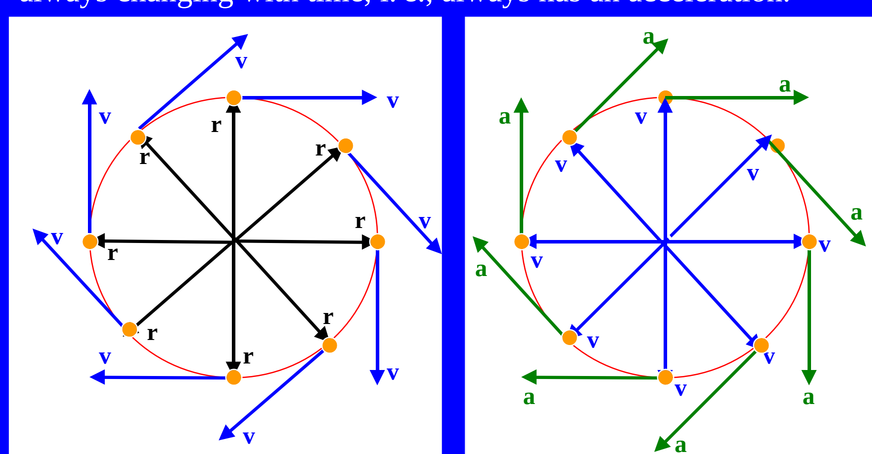
velocity on the equator $v_E = 2\,\pi\, R_E\, f_E$ = 464 m/s



# Circular Motion

**Centripetal Acceleration**

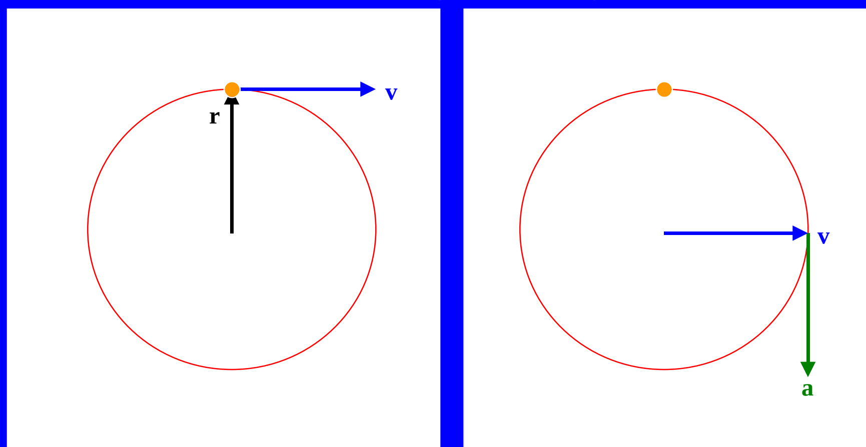
• The velocity of an object undergoing uniform circular motion is always changing with time, i. e., always has an acceleration.



## Circular Motion

### Centripetal Acceleration

• Just like the velocity **v** is the rate of change of position **r**, the acceleration **a** is the rate of change of velocity.

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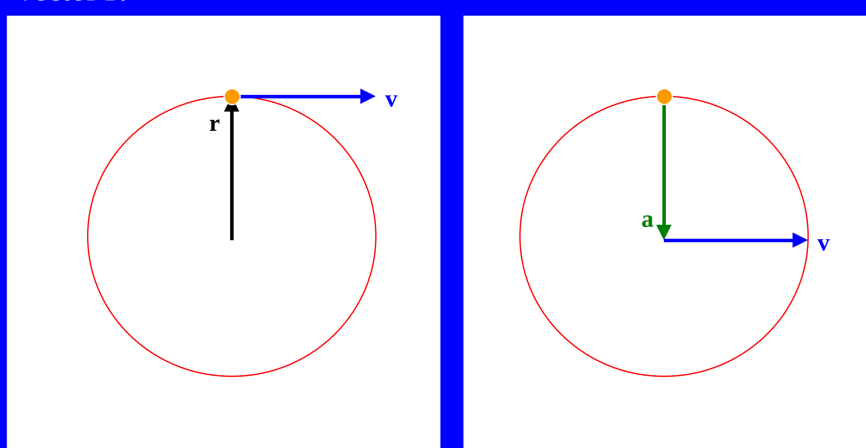
## Circular Motion

### Centripetal Acceleration

• The direction of the acceleration **a** is the opposite of the position vector **r**.

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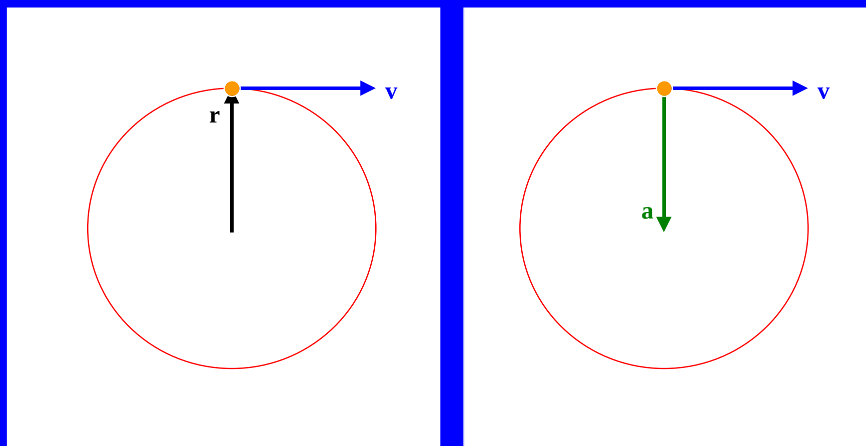
# Circular Motion

## Centripetal Acceleration --- Direction

• Therefore, the acceleration is always pointing to the center of the circle, i. e., centripetal acceleration.

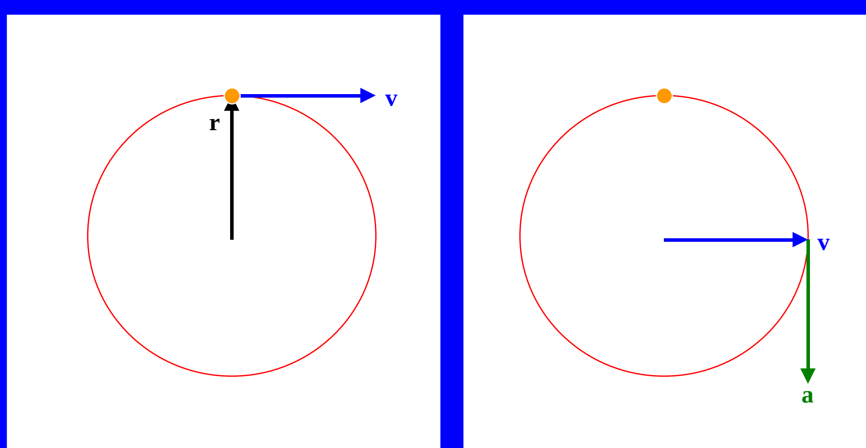





# Circular Motion

## Centripetal Acceleration --- Magnitude

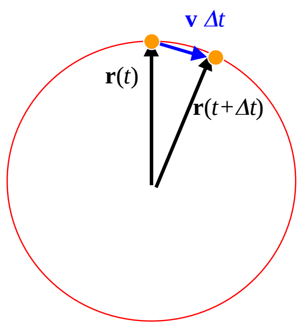
• By direct analogy, $v/r = a/v$ , so $a = v^2/r$



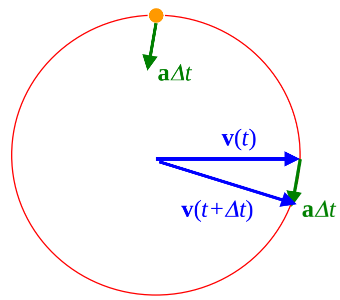
# Circular Motion

**Centripetal Acceleration --- Magnitude**

• By direct analogy, $v/r = a/v$ , so $a_c = v^2/r = r\,(2\pi/T)^2$




$$\mathbf{v} = \frac{\mathbf{r}(t+\Delta t) - \mathbf{r}(t)}{\Delta t}$$




$$\mathbf{a} = \frac{\mathbf{v}(t+\Delta t) - \mathbf{v}(t)}{\Delta t}$$

Do 5.CQ.002



# Circular Motion

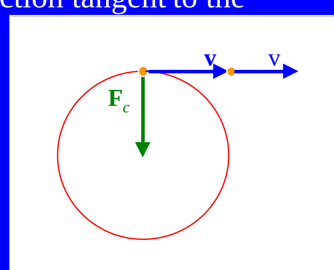
**Centripetal Force**

• By Newton's Second Law $\Sigma\mathbf{F} = m\,\mathbf{a}$ , there is a centripetal force with a direction parallel to the centripetal acceleration, and a magnitude $F_c = m\,a_c = m\,v^2 / r$

• Any object undergoing circular motion needs a centripetal force to maintain the circular motion.

• If the centripetal force is suddenly cut off, the object will continue moving with a constant velocity with a direction tangent to the circular path.

• E. g., a passenger inside in turning car feels like being pull away because of the tendency of moving in the tangent direction, until being supported by something that provides the centripetal force.



## Circular Motion

**Centripetal Force and Safe Driving (Example 7)**

• Static friction provides the centripetal force on a level road for a car to make a turn of radius $r$: $F_c = m\, v^2 / r \le \mu_s\, mg$ , or

$$v \le \sqrt{\mu_s g r}$$

• Can only turn with smaller speed for smaller $\mu_s$ (more slippery road or worn out tires) or for smaller $r$ (tighter turn).

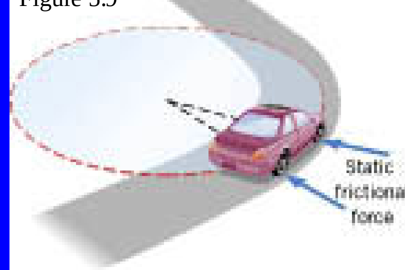


Figure 5.9

Do Concept Simulation 5.2 , 5.CQ.006, 008



## Circular Motion

**Centripetal force of a turning airplane**

• An airplane has to tilt its wings to provide the centripetal force by the horizontal component of the lifting force to make a turn of radius $r$: $F_c = m\, v^2 / r = L \sin\theta$ , and $mg = L \cos\theta$ , or

$$\tan\theta = \frac{v^2}{rg}$$

• Has to tilt more for higher speed or for smaller turning radius.

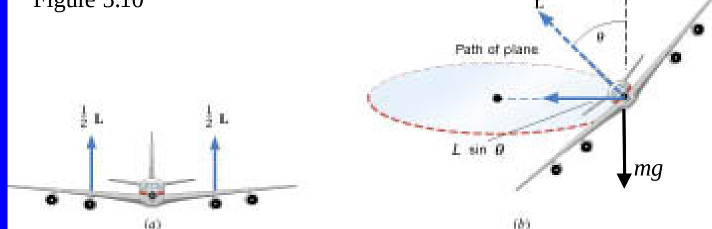


Figure 5.10

## Circular Motion

### Centripetal force on a banked curve

• The situation on a banked curve is similar, except now the normal force from the road is tilted: $F_c = m\,v^2 / r = F_N \sin\theta$, and $mg = F_N \cos\theta$, or

$$\tan\theta = \frac{v^2}{rg}$$

• Banking at larger angle for higher speed or for smaller turning radius. Or, has to turn with larger $r$ for higher speed for a fixed $\theta$.

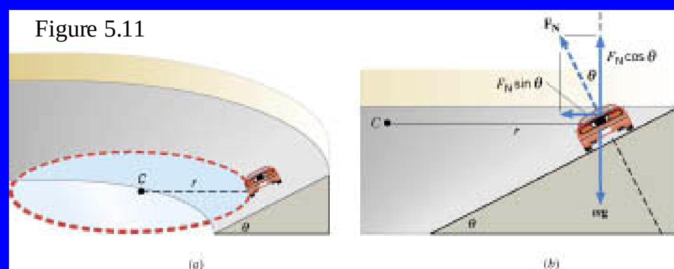


Figure 5.11



## Circular Motion

### Centripetal force of a satellite in circular orbit

• The the centripetal force is provided by gravity: $F_c = m\,v^2 / r = GmM_E/r^2 =$, or

$$v = \sqrt{\frac{GM_E}{r}} = \frac{2\pi r}{T}$$

• For synchronous satellites, $T$ = 24 hours,

$$r = (GM_E)^3 \left(\frac{T}{2\pi}\right)^{2/3}$$

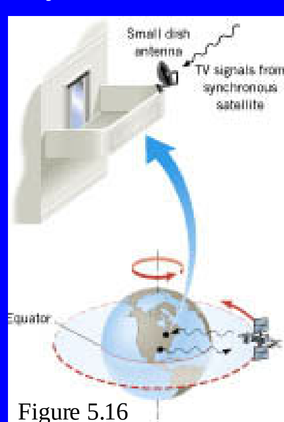


Figure 5.16



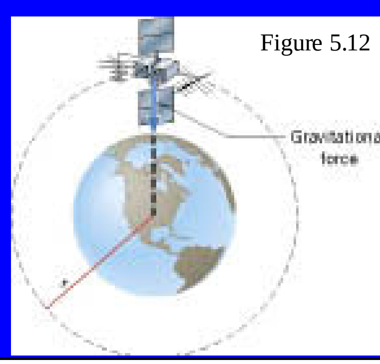


Figure 5.12

## Circular Motion

**Apparent weightlessness inside a satellite in circular orbit**

• The the centripetal force is provided by gravity:

$F_c = m\,v^2 / r = GmM_E/r^2 = mg$ , with the direction of the centripetal acceleration the same as **g** (pointing to the center of the Earth).

• Remember that the apparent weight inside the satellite is

$\mathbf{W}' = m\,(\mathbf{g} - \mathbf{a}) = 0$ , i. e., apparent weightlessness (same as free fall).

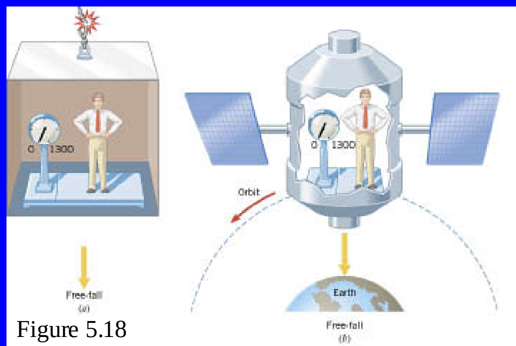



Figure 5.18



## Circular Motion

**Artificial Gravity**

• It is possible to produce artificial gravity in an apparent weightlessness state that might be harmful for long-term human living. The idea is to live in the inside surface of a rotating cylinder. Then, there is an apparent weight pointing outwards with a magnitude $W' = m\,a_c = m\,v^2 / r$ , or the apparent gravitational acceleration is $g' = v^2 / r = r\,(2\pi/T)^2$.

• To get a value of $g' = 10$ m/s$^2$ for a space station with a radius $r = 1$ km, a velocity $v = 100$ m/s, or a period $T = 62.8$ s is needed.

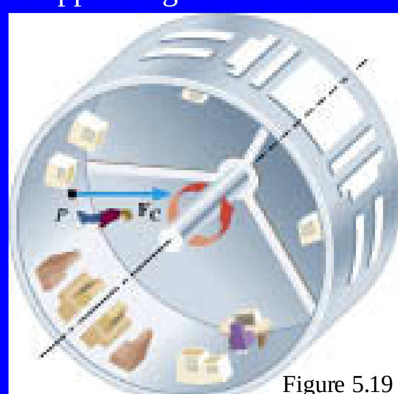



Figure 5.19

## Circular Motion

**Vertical Circular Motion**

- Centripetal force is the sum of the normal force and the weight of the object along the direction pointing to the center.
- At the top (position 3), both the normal force and the weight of the object are pointing downwards to the center, so $F_c = m\,v^2 / r = mg + F_N$, require $v \geq \sqrt{rg}$
- At the bottom (position 1), the normal force is pointing upwards and the weight is pointing downwards to the center, so $F_c = m\,v^2 / r = F_N - mg$
- At positions 2 or 4, the normal force is pointing horizontally to the center, so $F_c = m\,v^2 / r = F_N$

Do 5.CQ.013, 014

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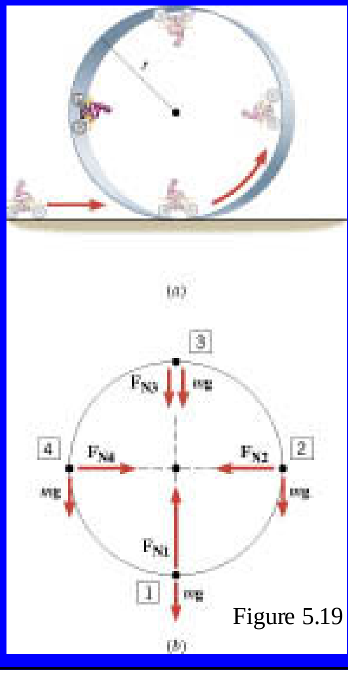


Figure 5.19