

Physics of Rotation

Why learn the physics of rotation?

- The motion of many objects in the universe involve rotation.
- Uniform circular motion is a special case of rotation.
- May have non-circular path and changing speed.
- Can be understood easily with an analogy with straight line motion.
- First learn to describe rotation (rotational kinematics), then learn how rotation is produced by forces/torques (rotational dynamics).

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Physics of Rotation

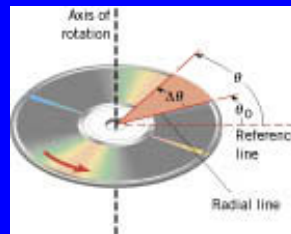
| Rotational Motion | Linear Motion |
|--|--|
| Angular displacement θ | Displacement x |
| Angular velocity ω | Velocity v |
| Angular acceleration α | Acceleration a |
| Rotational kinematics: | Linear kinematics: |
| $\omega = \omega_0 + \alpha t$ | $v = v_0 + at$ |
| $\theta = \frac{1}{2}(\omega_0 + \omega)t$ | $x = \frac{1}{2}(v_0 + v)t$ |
| $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ | $x = v_0 t + \frac{1}{2}at^2$ |
| $\omega^2 = \omega_0^2 + 2\alpha\theta$ | $v^2 = v_0^2 + 2ax$ |
| Torque τ | Force F |
| Moment of inertial I | Mass m |
| Newton's 2 nd law: $\tau = I\alpha$ | Newton's 2 nd law: $F = ma$ |
| Kinetic energy $\frac{1}{2}I\omega^2$ | Kinetic energy $\frac{1}{2}mv^2$ |
| Angular momentum $L = I\omega$ | Linear momentum $p = mv$ |

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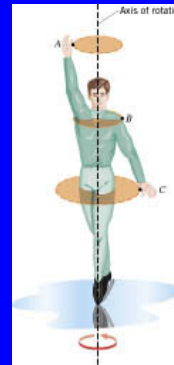
Rotational Kinematics

Angular Displacement

- “When a rigid body rotates about a fixed axis, the angular displacement is the angle $\Delta\theta$ swept out by a line passing through any point on the body and intersecting the axis of rotation perpendicularly.”
- Can be positive (counterclockwise) or negative (clockwise).
- Similar to “the number of revolutions” the object spins.
- Analogous to a component of the displacement vector.
- Angular displacement can be non-zero even if a point goes back to its original position. In this case, $\theta = n 2\pi$ ($n = \text{integer}$)
- SI unit: radian (rad). Other units: degree, revolution.



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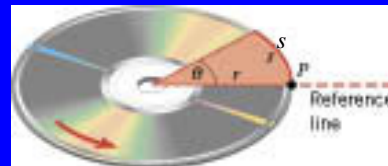


Rotational Kinematics

Radian: a unit of angle

- Definition:

$$\theta \text{ (in radians)} = \frac{\text{Arc length } s}{\text{Radius } r} = \frac{s}{r}$$



- Angle is a dimensionless quantity since it is a ratio.
- Radian is a dimensionless unit of angle (with a value of 1).
- Conversion with revolution: $1 \text{ revolution} = 2\pi \text{ radians}$
- Conversion with degree: $360^\circ = 2\pi \text{ radians}$, so

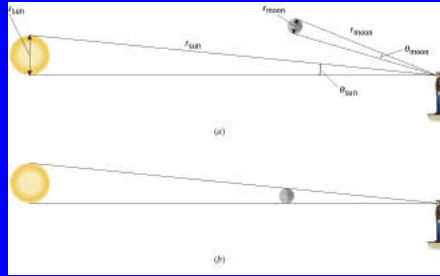
$$1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

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Rotational Kinematics

Example 2: Arc length and Angle

- Sun-Earth distance: 1.5×10^{11} m
- Moon-Earth distance: 3.85×10^8 m
- Solar diameter: 1.4×10^9 m
- Lunar diameter: 3.48×10^6 m



- angle:

$$\theta_S \approx \frac{d_S}{r_{S-E}} = \frac{1.4 \times 10^9 \text{ m}}{1.5 \times 10^{11} \text{ m}} = 9.3 \times 10^{-3} \text{ rad} = 0.53^\circ$$

$$\theta_M \approx \frac{d_M}{r_{M-E}} = \frac{3.48 \times 10^6 \text{ m}}{3.85 \times 10^8 \text{ m}} = 9.0 \times 10^{-3} \text{ rad} = 0.52^\circ$$

- Total eclipse possible (see Figure 8.5).

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Rotational Kinematics

Angular Velocity

- Average angular velocity, is defined by

$$\bar{\omega} = (\text{angular displacement}) / (\text{elapsed time}) = \Delta\theta / \Delta t .$$

- SI unit of angular velocity, is radian per second (rad/s).

- **Instantaneous angular velocity**, is defined by

$$\omega(t) = \lim_{\Delta t \rightarrow 0} \frac{\theta(t + \Delta t) - \theta(t)}{\Delta t} ; \quad \text{or} \quad \omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t}$$

- Basic application: $\Delta\theta = \bar{\omega} \Delta t .$
- Angular velocity can be positive or negative.
- For uniform circular motion, $\omega = 2\pi / T = 2\pi f$, so ω is also called angular frequency.

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Rotational Kinematics

Angular Acceleration

- **Average angular acceleration**, is defined by
$$\bar{\alpha} = (\text{change in angular velocity})/(\text{elapsed time}) = \Delta\omega / \Delta t .$$
- SI unit of angular acceleration: radian per second squared (rad/s²).

- **Instantaneous angular acceleration**, is defined by

$$\alpha(t) = \lim_{\Delta t \rightarrow 0} \frac{\omega(t + \Delta t) - \omega(t)}{\Delta t}; \quad \text{or} \quad \alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t}$$

- Basic application: $\Delta\omega = \omega - \omega_0 = \bar{\alpha} \Delta t$. For constant α motion,
$$\Delta\theta = \bar{\omega} \Delta t = (\omega + \omega_0) \Delta t / 2 = \omega_0 \Delta t + \alpha \Delta t^2 / 2$$
- Angular acceleration can be positive or negative.

Do 8.CQ.002, 003

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The Equations of Rotational Kinematics for constant α

| Rotational Motion | Linear Motion |
|---|-------------------------------|
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| Rotational kinematics: | Linear kinematics: |
| $\omega = \omega_0 + \alpha t$ | $v = v_0 + at$ |
| $\theta = \frac{1}{2}(\omega_0 + \omega)t$ | $x = \frac{1}{2}(v_0 + v)t$ |
| $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ | $x = v_0 t + \frac{1}{2}at^2$ |
| $\omega^2 = \omega_0^2 + 2\alpha\theta$ | $v^2 = v_0^2 + 2ax$ |

- two independent equations, five quantities: $\theta, \alpha, \omega_0, \omega, t$
- most convenient to use the equation that has the three known quantities and the unknown quantity that is being asked for.

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Rotational Kinematics

Angular Variables and Tangential Variables

- For an object rotating with an instantaneous angular velocity ω and an instantaneous angular acceleration α , any point on the object at a distance r from the rotating axis has a tangential velocity v_T and tangential acceleration a_T , given by:

$$v_T = r\omega, \quad a_T = r\alpha$$

- This is because $\Delta\theta = \frac{\Delta s}{r}$, so

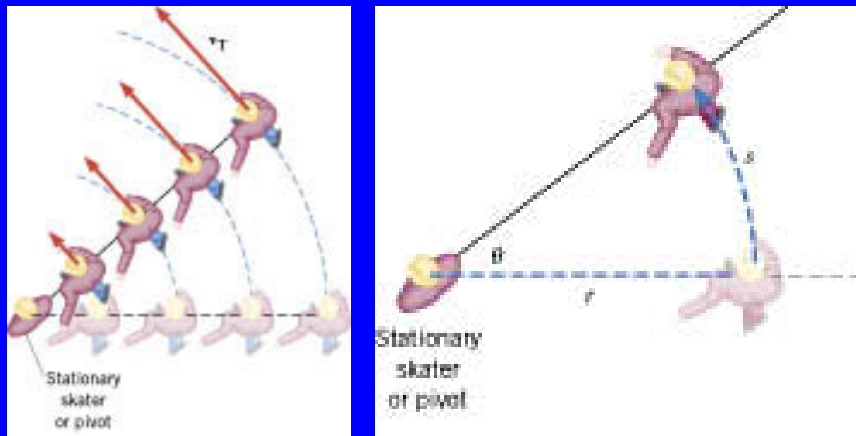
$$v_T = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{r\Delta\theta}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = r\omega$$

$$a_T = \lim_{\Delta t \rightarrow 0} \frac{\Delta v_T}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{r\Delta\omega}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = r\alpha$$

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Rotational Kinematics

Angular Variables and Tangential Variables (Example)



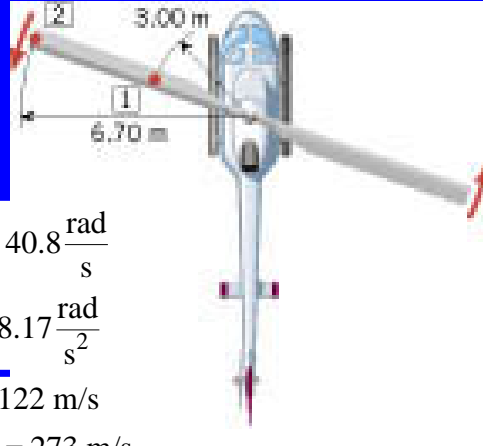
- The skater skating with the largest radius r has to move and accelerate the fastest.

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Rotational Kinematics

Angular Variables and Tangential Variables (Example 6)

- $\omega = 6.50 \text{ rev/s}$
- $\alpha = 1.30 \text{ rev/s}^2$
- At point 1, $r_1 = 3 \text{ m}$
- At point 2, $r_2 = 6.7 \text{ m}$



$$\omega = 6.5 \frac{\text{rev}}{\text{s}} = 6.5 \frac{\text{rev}}{\text{s}} \frac{2\pi \text{ rad}}{\text{rev}} = 40.8 \frac{\text{rad}}{\text{s}}$$

$$\alpha = 1.3 \frac{\text{rev}}{\text{s}^2} = 1.3 \frac{\text{rev}}{\text{s}^2} \frac{2\pi \text{ rad}}{\text{rev}} = 8.17 \frac{\text{rad}}{\text{s}^2}$$

$$v_{1T} = r_1 \omega = (3 \text{ m})(40.8 \text{ rad/s}) = 122 \text{ m/s}$$

$$v_{2T} = r_2 \omega = (6.7 \text{ m})(40.8 \text{ rad/s}) = 273 \text{ m/s}$$

$$a_{1T} = r_1 \alpha = (3 \text{ m})(8.17 \text{ rad/s}^2) = 24.5 \text{ m/s}^2$$

$$a_{2T} = r_2 \alpha = (6.7 \text{ m})(8.17 \text{ rad/s}^2) = 54.7 \text{ m/s}^2$$

Do 8.CQ.004, 008, 009, 013