Interaction of Shear-Alfvén Wave Packets: Implication for Anisotropic MHD Turbulence in the Interstellar Medium

C. S. Ng and A. Bhattacharjee
Department of Physics and Astronomy, The University of Iowa

Weak magnetohydrodynamic turbulence in the presence of a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ is dominated by three-wave interactions that mediate the collisions of shear-Alfvén wave packets propagating in opposite directions parallel to the magnetic field. We give a detailed analytical and numerical treatment of such interactions. Using the ideal MHD equations, it is shown that 3-wave interactions are generally nonzero if the $k_z = 0$ Fourier components of the wave-packets are nonzero. From the reduced MHD equations, we calculate in closed form the 3-wave and 4-wave interaction terms, and show the latter to be generally asymptotically subdominant. From the analytical expressions, the scaling of three-wave couplings is calculated by asymptotic analysis and a direct numerical evaluation of the nonlinear interaction. A new relation is derived between the spectral index of three-wave coupling and the spectral indices of two random-amplitude wave packets. This relation has significant implications for the anisotropic energy spectrum which, by heuristic scaling arguments, is found to be similar to the 2D spectrum in hydrodynamics.